Bounds on Solar Neutrino Packet Lengths

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(Dated: March 23, 2020)

Abstract

In 1976 S. Nussinov [1] made the original proposal that solar neutrino packet length could be constrained by the β^+-decay interval over which the neutrino is emitted. That packet length in turn could be used to estimate a coherence length beyond which the neutrino mass states, propagating with slightly different velocities due to their squared mass splitting, would separate and no longer be detectable coherently, i.e., flavor oscillations would no longer be possible. In this article we examine the Nussinov calculations, the solar plasma physics, weak interaction details and the packet length known from terrestrial reactor experiments. We conclude that solar neutrino packet lengths are not constrained by the β^+-decay interval, but lower bounded by the lifetime of the virtual W^+ boson in the weak interaction process and nominally by the necessary packet length observed in terrestrial reactor oscillations.

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I. INCOHERENCE LENGTH

To estimate the distance at which a solar neutrino packet becomes incoherent, we require an estimate of the neutrino packet length σ_x (the spatial width of, say, a Gaussian packet along the Cartesian axis of propagation) in the following [2]:

$$L_{coh} = \frac{\sigma_x}{\Delta v} = \sigma_x \frac{2E^2}{\Delta m^2} \tag{1}$$

II. NUSSINOV 1976 INTERRUPTED PACKET EMISSION

In 1976 Nussinov [1] proposed that the neutrino packet length is approximately the time interval during which it is emitted by the decaying nucleus, multiplied by the speed of light (the neutrino being ultrarelativistic), i.e.,

$$\sigma_x = c \cdot \tau_{emission}$$

He then reasoned that because solar neutrinos are emitted from nuclei in hot, dense plasma, the neutrino packet emission interval is shortened through collisions of the emitter with neighboring nucleons or nuclei while the emission is in progress. This concept originated in the context of stellar spectroscopy, A. A. Michelson hypothesizing in 1895 that causes for the observed broadening of spectral lines included "...limitation of the number of regular vibrations by more or less abrupt changes of phase amplitude or plane of vibration, caused by collisions." [3]. Discontinuity in the emission or absorption of the photon sine wave train introduces components of higher frequency and thereby broadens a spectral line [4] (the frequency distribution of a clipped sinusoidal wave train is inversely proportional to the number of cycles included in the pulse [5]).

At the approximate surface of the Sun $r_\odot = 1$ (nominal photosphere temperature 5772 K [6]), there is an appreciable number of neutral or partially ionized atoms capable of absorbing optical radiation[1], creating the dark lines that identify particular elements in the solar spectra, Gustav Kirchhoff writing in 1860 that "chemical analysis of the sun's atmosphere requires only the examination of those substances which, when brought into a flame, produce bright lines which coincide with the dark lines of the solar spectrum"[8]. By *optical radiation* we refer to photons absorbed (or emitted) during electron transitions between an upper and

[1] See §14 Ionization and §17 Opacity in [7].

lower energy level in an atom, typically $E1$ electric dipole radiation [9]. It is reasonable to expect that the electron orbitals of atoms in gas may be disturbed by collisions between the atoms, or by collisions of free electrons with atoms, modifying the light emissions and thereby broadening the associated spectral line [9].

Neutrinos, however, are not produced in the photosphere, but rather in the hot, dense ionized plasma of the core of the Sun (see plasma density-temperature regime chart Figure 20.1 in [10]), the "emission regions occurring in a sequence of shells [which overlap, all ending by solar radius $\sim 0.3\, r_\odot$], following closely the location of nuclear reaction, orderly arranged in a sequence dependent on their temperature" [11]. Nussinov estimated the nearest neighbor distance there, i.e., plasma protons near the neutrino-emitting nucleus, to be $d \approx 10^{-9}$ cm (and defines that to be equivalent to $\approx \pi$ phase between nucleus and neighbor particles) and the proton thermal velocity to be $v_{therm} \approx 10^{-3}$ c (or 3×10^7 cm/s). Using those values, he obtained an estimated collision interval (time between collisions of protons with β^+-decaying nuclei)

$$\tau_{col} = \frac{d}{v} = \frac{10^{-9}\,\text{cm}}{3\ \times 10^7\,\text{cm/s}} \approx 3\ \times 10^{-17}\,\text{s} \tag{2}$$

Tracing his route to the collision estimate in Eq. (2), he gives us the matter density as $\approx 100\,\text{g}\,\text{cm}^{-3}$. If we assume primarily ionized hydrogen (i.e., protons) present we can multiply the matter density by Avogadro's number to obtain a proton number density, $N_p \approx 100\,\text{g}\,\text{cm}^{-3} \times 1\,\text{mol}\,\text{H}/1\text{g} \times 6 \times 10^{23}\,\text{entities/mol} \approx 6 \times 10^{25}\,p/\text{cm}^3$. Each proton (and the small number of heavier nuclei present) has therefore a $1/N_p$ "cell" of about $V = 1.6 \times 10^{-26}\,\text{cm}^3$ volume. Setting that equal to the volume of a sphere, $4\pi r^3/3$, we can solve for the radius and obtain an approximate distance between plasma particles[2]:

$$\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
r &= \left(\frac{3}{4\pi}V\right)^{1/3} \\
r &= \left(\frac{3}{4\pi}1.6\ \times 10^{-26}\,\text{cm}^3\right)^{1/3} \sim 1.5\ \times 10^{-9}\,\text{cm}
\end{aligned}$$

Interestingly, Chandrasekhar's 1943 average nearest neighbor distance formula [12] produces about the same result

$$D = \Gamma\,(4/3)\left(\frac{3}{4\pi}\right)^{1/3} N_p^{-1/3} = 0.55396 N_p^{-1/3} \sim 1.4\ \times 10^{-9}\,\text{cm} \tag{3}$$

[2] See, e.g., equation (14.43) in [7].

Nussinov provided the plasma temperature at the location of interest as $k_B T \approx 1\,\mathrm{keV}$ ($11.6\ \times 10^6\,\mathrm{K}\ \times k_B = 1\,\mathrm{keV}$, using the Boltzmann constant k_B in units $k_B = 8.6173\ \times 10^{-5}\,\mathrm{eV\,K^{-1}}$). With that temperature you can calculate the proton thermal velocity working in natural units eV, i.e., $v_{therm} = \sqrt{k_B T/m} = \sqrt{T_e/m}$ [13] where T_e is the temperature of the plasma particle in electron volts ($\sim 10^3\,\mathrm{eV}$ in this case) and $m \approx 10^9\,\mathrm{eV/c^2}$ is roughly the mass of a proton ($938\,\mathrm{MeV/c^2}$ to be exact). That gives you quickly $v_{therm} = \sqrt{(10^3\,\mathrm{eV})/(10^9\,\mathrm{eV/c^2})} = 10^{-3}\,\mathrm{c}$, the velocity as a fraction of the speed of light c.

So, using his parameters and methods we reproduce his estimate that the neutrino-emitting nuclei are being hit about every $3\ \times 10^{-17}$ s by plasma protons. Inverting that collision interval gives the rate of collsions, $\nu = 1/\tau_{col} = 1/(3\ \times 10^{-17}\ \mathrm{s}) \approx 3\ \times 10^{16}\ \mathrm{s}^{-1}$. For comparison, we have Bahcall [14] (citing in turn A. Loeb *Phys. Rev. D 39, 1009 (1989)*) giving an estimate of $10^{15}\ \mathrm{s}^{-1}$ collision rate on ^{7}Be ions in the solar interior[3], within an order of magnitude of our figure.

A. Coulomb potential

The question is, do any of those collisions actually interrupt a neutrino emission? Nussinov reasons that the Coulomb potential felt by the $^7_4$Be nucleus from a neighboring proton is $4e^2/r \approx 500$ eV and changes by a factor of two in a collision (because he defined the neighbor distance to be equivalent phase $\sim \pi$ and considers a phase change $\geq \pi/2$ to constitute an interruption).

The Coulomb potential arising from a neighbor proton Z_1 within d distance of a $^7_4$Be nucleus Z_2 is:

$$E_C = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{d} \tag{4}$$

(Refer to Eq.(14.6) in [15] and Eq.(20) in [16], $d \geq R_C$, i.e., distance of approach greater than or equal to the Coulomb charge radius of the target nucleus.) For convenience we will set $\hbar = c = \epsilon_0 = 1$ and work in natural units, all quantities expressed in eV. In this case the elementary charge is $e = \sqrt{4\pi\alpha} \approx 0.303$ dimensionless. We convert our distance $d \approx 10^{-9}$ cm to natural unit length $d = 10^{-9}\ \mathrm{cm}/(\hbar c) = 10^{-9}\ \mathrm{cm}/(6.5821\ \times 10^{-16}\,\mathrm{eV\,s}\ \times 2.9979\ \times 10^{10}\,\mathrm{cm/s}) = 5.0677\ \times 10^{-5}\,\mathrm{eV}^{-1}$.

[3] Bahcall does not specify what species of plasma particles are considered in calculating that rate.

Substituting the proton charge $Z_1 = 1$, ${}^7_4$Be charge $Z_2 = 4$ and other quantities in Eq. (4) we calculate:

$$\begin{aligned}
E_C &= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{d} \\
&= \frac{1 \cdot 4 \cdot (0.303)^2}{4\pi}\frac{1}{5.0677 \times 10^{-5}\,\mathrm{eV}^{-1}} \\
&= \frac{0.3672}{6.3683 \times 10^{-4}\,\mathrm{eV}^{-1}} \\
&\approx 576 \text{ eV}
\end{aligned}$$

Nussinov then states that the resulting phase change of the emitting nucleus during the collision is $\Delta\phi \approx \Delta E \Delta t/h \approx 12$ rad, much greater than the limit of $\approx \pi/2$ said to constitute interruption of an emission by phase loss of the emitter [4]. Using his phase change relation, collision time and his stated factor of two change in the potential we approximate his result:

$$\Delta\phi \approx (\Delta E)(\frac{\Delta t}{h}) = (2)(576 \text{ eV})(\frac{3 \times 10^{-17}\text{ s}}{4.1357 \times 10^{-15}\,\mathrm{eV\,s}}) \sim 8.4 \text{ rad}$$

He concludes that the plasma proton collisions at interval $\Delta t = 3 \times 10^{-17}$ s are more than energetic enough to interrupt a neutrino packet emission and therefore constrain that packet length to: $\sigma_x = c \cdot 3 \times 10^{-17}\text{ s} \sim 9 \times 10^{-7}$ cm or less.

B. Resulting incoherence length

Using equation Eq. (1) and the 1976 Nussinov neutrino mass split Δm^2 of $\sim 1\text{ eV}^2$ with neutrino energy ~ 1 MeV and packet length $\sim 9 \times 10^{-7}$ cm, the coherence length is ≈ 18 km (he estimated $\approx 1 - 10\,\mathrm{km}$), i.e., the neutrino would become incoherent within a few kilometers, so of course could not arrive coherently at Earth:

$$\begin{aligned}
L_{coh} &= \sigma_x \frac{2E^2}{\Delta m^2} \\
&= 8.99 \times 10^{-7}\text{ cm} \cdot \frac{2 \cdot 1\text{ MeV}^2}{1\text{ eV}^2} \\
&= 8.99 \times 10^{-9}\text{ m} \cdot 2 \times (10^6)^2 \\
&= 17.98 \text{ km}
\end{aligned}$$

[4] See also Collins discussing the Weisskopf criterion of 1 radian phase change of emitter constituting an interruption in an emitted photon wave train.[4]

We can update his calculation using a recent value of $\Delta m_{21}^2 = 7.5 \times 10^{-5}$ eV^2 and a neutrino energy of 0.862 MeV. That energy represents the higher monoenergetic line (emitted neutrino energy in the beta-decay) from the transition of ^{7}Be to the ground state of ^{7}Li in a primarily continuum electron capture [14].

Using the same neutrino packet length calculated above, but with the updated mass split and neutrino energy, we obtain a coherence length of $\sim 178 \times 10^3$ km with equation Eq. (1). The distance to Earth is 1.496×10^8 km so we would again expect only decoherent ^{7}Be neutrinos to arrive at Earth detectors. The coherence distance is directly proportional to the neutrino energy per Eq. (1), so solar species of lower energy would also arrive at Earth decoherent.

III. PLASMA NUCLEAR CONSIDERATIONS

While the above approach Section II represents a very clever and original insight by Nussinov (in suggesting that neutrino mass states may separate and no longer overlap over suitable distances), the mechanism constraining the supposed time available for emission of the neutrino packets assumes that elastic scattering processes, e.g., plasma proton collisions with the target β^+-decay candidate nucleus, can interrupt an essentially nuclear process, i.e., a process involving nuclear energy levels and spatial scales. To "see" the nucleus in an atom requires a scattering probe (a particle projectile colliding with the target atom) de Broglie wavelength of $\lambda \approx 10\,\mathrm{fm}$ [17]. To interact with a nucleon within the nucleus requires $\lambda \approx 1\,\mathrm{fm}$ [17]. Using the plasma temperature from above, 11.6 MK, we see the average proton kinetic energy is [18]:

$$\langle E_p \rangle = \frac{3}{2} k_B T = \frac{3}{2}(8.6173 \times 10^{-5}\mathrm{eV/K})11.6 \times 10^6\,\mathrm{K} \sim 1.5\,\mathrm{keV}$$

At that kinetic energy a plasma proton would have a de Broglie wavelength of:

$$\lambda = \frac{h}{p} = c\frac{h}{\sqrt{2m_p E_p}} = 2.9979 \times 10^8\,\mathrm{m/s}\frac{4.1357 \times 10^{-15}\,\mathrm{eV\,s}}{\sqrt{(2)(938 \times 10^6\,\mathrm{eV})(1.5 \times 10^3\mathrm{eV})}} \simeq 739\,\mathrm{fm}$$

We used the Newtonian relation $p = \sqrt{2mE}$ to obtain the non-relativistic (the corresponding velocity is only $\sim 0.002c$) momentum from the kinetic energy. A projectile of this wavelength would be unable to interact significantly with a nucleus, much less a proton within a nucleus.

But what about the Coulomb interaction we examined above? A change of 576 eV electrostatic field neighbor potential might constitute sufficient perturbation of an *atom* in the process of radiating or absorbing a photon, that is, one of the electrons of the atom's electron shells emitting or absorbing a photon in a transition from or to an excited level. The atomic energy transition levels involved are of the order $\mathcal{O}(\mathrm{eV})$. For example, the photospheric 249.7 nm B I (a neutral boron atom near the apparent surface of the Sun, the photosphere, where the temperature is low enough) line involves the absorption of a photon of 4.9639 eV kicking the lone outer shell $2p$ electron (with term $^2\mathrm{P}^{\circ}_{3/2}$, already in an excited state) up to $3s$ (with term $^2\mathrm{S}_{1/2}$) [5].

Contrast that level of energy with the Coulomb barrier of roughly 2 MeV that a plasma proton would have to overcome to penetrate to the nucleons within a $^8\mathrm{B}$ target nucleus:

$$
\begin{aligned}
V_c &= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{R_c} \\
&= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{1.1\left(m_A^{1/3}+m_B^{1/3}\right)} \\
&= \frac{1\cdot 5\left(\sqrt{4\pi\alpha\,(\hbar c)}\right)^2}{4\pi(1)}\frac{1}{(3.3047\,\mathrm{fm})} \\
&= \frac{5\,(0.303)^2\,(\hbar c)}{4\pi(3.3047\,\mathrm{fm})} = \frac{(0.459)(197.33\,\mathrm{MeV\,fm})}{41.527\,\mathrm{fm}} \\
&= 2.1787\,\mathrm{MeV}
\end{aligned}
$$

The $R_c = \left[1.1\left(m_A^{1/3}+m_B^{1/3}\right)\right]$ expression used the mass of the proton and $^8\mathrm{B}$ target nucleus in u, i.e., 1.00727 u for the proton and 8.021863 u for $^8\mathrm{B}$, to calculate the distance in fm between their two charge fields as they come almost into contact. The *atomic* mass given by Krane for $^8\mathrm{B}$ is 8.024606 u [15]. We subtracted the mass of the associated electrons, $5 \times 0.000549\,\mathrm{u} = 0.002743\,\mathrm{u}$, from the Krane atomic mass to give us the approximate mass of the *nucleus*, 8.021863 u. There is a small amount of electron binding energy, but it is of order $\mathcal{O}(10^{-6}\,\mathrm{u})$ so we forgo that correction.

The proton charge $Z_1 = 1$ and $^8\mathrm{B}$ charge $Z_2 = 5$. We used natural units for the vacuum permittivity $\epsilon_0 = 1$ and elementary charge $e = \sqrt{4\pi\alpha} \approx 0.303$ dimensionless. It was convenient to use $\hbar c$ in units MeV fm.

Even so, the protons do have a chance of quantum mechanical tunneling past the Coulomb barrier at higher relative collision velocities than the $\sim 0.002c$ mean plasma velocity we

[5] The 249.7 nm B I line is given in [19]. The transition data is from the NIST Atomic Spectra Database [20] A handbook of atomic spectroscopy is available there also [21].

calculated above. However, the Gamow peak energy (where the probability of success is highest) for this collision system at our plasma conditions given earlier is 20.8616 keV (calculated below). That energy is far out on the right skirt of the Maxwell-Boltzmann velocity distribution with equivalent velocity of $\sim 2.1211 \times 10^6$m/s, i.e., a very low rate of occurrence $\mathcal{O}(10^{-6})$. There is a small enhancement of cross section for a positive projectile incoming on a plasma ion with negative Debye shielding (a cloud of plasma electrons, the Debye sphere, which gather around positive ions), but for $Z_1 Z_2$ of order 10 or less it is only a few percent improvement at best [22].

IV. CALCULATE LIKELY BARRIER PENETRATION RATE

Let us try to quantify the possible rate of Coulomb barrier penetrations. We integrated the availability of projectiles vs energy over the relevant solar energy range 5 keV to 30 keV [22]. We made use of equation 3 in the NACRE II update of charged-particle-induced thermonuclear reaction rates for mass numbers $A < 16$ [16]:

$$N_A \langle \sigma v \rangle = 3.73 \times 10^{10}\, \hat{\mu}^{-1/2} T_9^{-3/2} \int_{0.005\text{ MeV}}^{0.030\text{ MeV}} E\sigma(E) \exp[-11.605 E/T_9] dE \tag{5}$$

where $T_9 = 0.0149$ is the Kelvin temperature of the plasma in units 10^9 K, E is the center of mass energy in units MeV, and $\hat{\mu} = m_A m_B/(m_A + m_B) = (1.00727 \cdot 8.021863)/(1.00727 + 8.021863) = 0.8949$ u the reduced mass of the target ${}^8_5$B nucleus and proton projectile collision system (see above for details of the ${}^8_5$B mass assignment).

The integral above in Eq. (5) is basically folding two distributions over the energy range of interest. One distribution, as we mentioned above, is the Boltzmann energy distribution of particles in a gas (see, e.g., equation (15) in Hans Bethe's 1967 Nobel lecture [23]), the availability of collision system participants *decreasing with increasing energy.* The second distribution is the cross section, which is proportional to the probability of penetrating the Coulomb barrier, that probability *increasing with energy* (there is an upper limit to the applicability of the astrophysical S-factor discussed below, but we will not encounter that is our context). The resulting curve (folding the two distributions) is a window of opportunity as it were, the Gamow window, centered around the Gamow peak energy, E_G [22]:

$$E_G = \left[(\pi\alpha Z_1 Z_2 kT)^2(\mu)\right]^{1/3} = 1.2204(Z_1^2 Z_2^2 \hat{\mu} T_6^2)^{1/3} \qquad \mu = \text{ reduced mass}, \hat{\mu} \text{ in atomic units}$$

For example, for the case at hand of the proton collision on ${}^{8}_{5}\mathrm{B}$ at $T_6 = 14.9420$ MK (this formula accepts plasma temperature in units 10^6 K and returns the Gamow peak energy in keV):

$$E_G = 1.2204(Z_1^2 Z_2^2 \hat{\mu} T_6^2)^{1/3} = (1.2204)\left[(1^2 \cdot 5^2)(0.89494\ \mathrm{u})(14.9420\ \mathrm{MK})\right]^{1/3} = 20.8618\ \mathrm{keV}$$

We constructed a simulated S-factor using the suggested rms (core) radius 2.84 fm of ^{8}B in the ground state [24]. We squared that radius and multiplied by π to create a simple geometrical cross sectional area approximation ($\sim 0.2534\,\mathrm{b}$).[6] Finally, we multiplied this cross section by the Gamow peak energy 20.8616 keV to simulate an S factor with value 5.2861 keV b. For comparison, the NACRE II rate for a similar reaction, $^9\mathrm{Be}(p,\gamma)^{10}\mathrm{B}$ (a plasma proton capture on the ^{9}Be ion) is ~ 1 keV b [16].

The S-factor is usually obtained by extrapolating experimental cross sections ($\sigma(E)$) obtained at relatively high energy to the low energies found in solar plasma, augmenting or replacing experimental data with calculations based on various nuclear models when needed:

$$S(E) = E\sigma(E)\exp[2\pi\eta(E)] \tag{6}$$

The Gamow penetration factor, $\exp[2\pi\eta(E)]$ in Eq. (6) above, describes the quantum mechanical tunneling probability. The cross section $\sigma(E)$ measured in the lab (e.g., with accelerator-driven collisions) is highly energy dependent and typically varies by orders of magnitude.

The S-factor is more slowly varying in the low-energy limit, so is used when calculating solar reaction rates. The S-factor is still energy dependent, but less so than the cross section, so for our rough approximations here we simply let S be a constant and include the energy dependent effects through the tunneling probability and the Boltzmann distribution within the integral in Eq. (5). We are not attempting to identify any resonances over our range of integration, but it seems unlikely that such a resonance would appear below our upper limit of 30 keV in any case.

We used our S-factor estimate to calculate the $E\sigma(E)$ quantity in the integrand in Eq. (5) dynamically for each energy step in the numerical integration process (computer code included in Appendix VII):

$$E\,\sigma(E) = S(E)\exp[-2\pi\,\eta(E)]$$

[6] This is rather crude, but keep in mind that a 20.8618 keV proton would have an effective cross-section of $\pi\lambda^2 \approx 31\,\mathrm{b}$, orders of magnitude larger than the nuclear target.

In that expression η, the Sommerfeld parameter (notice that the sign becomes negative when moved to the $S(E)$ side of equation Eq. (6) above), is:

$$\eta = 0.1575 Z_A Z_B (\hat{\mu}/E)^{1/2} \tag{7}$$

where $\hat{\mu} = m_A m_B/(m_A + m_B)$ is the reduced mass in atomic mass units of the target-projectile collision system, Z_A and Z_B the proton projectile and target ^{8}B nucleus charge respectively, and E is in MeV units (from the NACRE II paper cited). The exponential incorporating η, $\exp[-2\pi\eta(E)]$, is, as we mentioned earlier, the Gamow penetration factor, which describes the probability of quantum-mechanical tunneling through the Coulomb barrier, decreasing the cross section as energy decreases. This WKB approximation [7] for the Gamow penetration factor is valid if $2\pi\eta \geq 1$ [22].

At the Gamow peak energy for our collision system, using the parameter values specified, the tunneling probability (or Gamow penetration factor) is:

$$\exp[-2\pi\eta(20.8616\text{ keV})] = \exp[-2\pi\,(5.1578)] = 8.4256 \ \times 10^{-15}$$

$2\pi\eta$ in this case is $(2\pi)(0.1575)(1)(5)(0.8949\text{ u}/0.0208618\text{ MeV} = 32.4079$ using Eq. (7) above. The WKB approximation is therefore valid per the criterion above.

Integrating Eq. (5) produced $N_A\langle\sigma v\rangle = 9.0261 \ \times 10^{-13}$ cm^3 mol^{-1} s^{-1}. We divide that result by Avogadro's number N_A to obtain the reaction rate averaged over a Maxwell-Boltzmann distribution: $\langle\sigma v\rangle = 1.4988 \ \times 10^{-36}$ cm^3 s^{-1} for our proton impingement on ^{8}B.

For comparison, the NACRE II paper cited above gave for a similar reaction, ^{9}Be$(p,\gamma)^{10}$B (a plasma proton capture on the ^{9}Be ion) thermonuclear reaction rate at $T = 15 \times 10^6$ K the value $N_A\langle\sigma v\rangle = 1.43 \ \times 10^{-10}$ cm^3 mol^{-1} s^{-1}, which becomes $\langle\sigma v\rangle = 2.374 \ \times 10^{-34}$ cm^3 s^{-1} when divided by N_A. It appears that part of the increase in reaction rate for this ^{9}Be$(p,\gamma)^{10}$B system relative to our ^{8}B target can be attributed to the increase of the Gamow penetration factor to 6.6234×10^{-13}, the Coulomb barrier potential decreasing with one less proton than our ^{8}B nucleus target. Also, NACRE II integrated over an energy range extending past 1 MeV and included several resonances.

[7] Wentzel, Kramers, Brillouin and Jeffreys developed a quasi-classical method for approximating solutions to linear second-order differential equations like the Schrödinger equation. Gamow, Condon and Gurney used this method to calculate the probability of α particles tunneling through the nuclear Coulomb barrier.

To estimate the number of collisions per second we multiply the reaction rate times the number density of the ^{8}B target and proton projectiles:

$$f_{col} = N_{(^{8}\mathrm{B})} N_p \langle \sigma v \rangle \tag{8}$$

Having no data on the estimated dynamic abundance of the ^{8}B target nucleus, we use the mass fraction of precursor ^{3}He at radius 0.0460 in the B16 GS98 data set to calculate the number density of that nucleus:

$$\begin{aligned}
{}^{3}\mathrm{He\,mass\;fraction}(r_\odot = 0.0460) &= 1.3440 \times 10^{-5} \\
\mathrm{matter\;density}(r_\odot = 0.0460) &= 130.1\,\mathrm{g\,cm^{-3}} \\
{}^{3}\mathrm{He\;\;density}(r_\odot = 0.0460) &= (1.3440 \times 10^{-5})(130.1\,\mathrm{g\,cm^{-3}}) = 1.7485 \times 10^{-3}\,\mathrm{g\,cm^{-3}} \\
{}^{3}\mathrm{He\,number\;density}(r_\odot = 0.0460) &= [(1.7485 \times 10^{-3}\,\mathrm{g\,cm^{-3}})/(3.016029\,\mathrm{g})](N_A) = 3.49 \times 10^{20}\ \mathrm{cm^{-3}}
\end{aligned}$$

N_A above is Avogadro's number, $6.0221 \times 10^{23}\,\mathrm{mol^{-1}}$. We obtain the number density of ^{3}He in line 4 of the equation group above by dividing the mass density of the nucleus by its molar mass, 3.016029 g. The molar mass is equivalent to the atomic weight in u, where u is the unified atomic mass unit defined as 1/12 the mass of a ^{12}C atom.

About 16.7% of the 3.49×10^{20} cm^{-3} ^{3}He available at that radius, 5.8306×10^{19} cm^{-3}, fuses with available ^{4}He (α particles) to produce ^{7}Be. A minute fraction of that, $0.12\% = 6.9968 \times 10^{16}$ cm^{-3}, captures a proton to form ^{8}B via $^{7}\mathrm{Be} + p \rightarrow {}^{8}\mathrm{B} + \gamma$, our proton bombardment target (fusion chain fractions from [25], their figure 4).

Thus we approximate the ^{8}B number density as 6.9968×10^{16} cm^{-3}. We use proton density equal to the electron density we obtained from the B16 GS98 SSM data i.e., 5.4670×10^{25} cm^{-3}. Multiplying our reaction rate, $\langle \sigma v \rangle = 1.4988\ \times 10^{-36}$ cm^{3} s^{-1}, times the proton and ^{8}B number densities per Eq. (8) above) we obtain a likely barrier-penetrating collision rate of $f_{col} = 5.7332 \times 10^{6}$ s^{-1}, or a collision interval of $1/f_{rate} = 1.7442 \times 10^{-7}$s.

If that interval was constraining neutrino emission it would imply neutrino packets $\sigma_x = c \cdot 1.7442\ \times 10^{-7}\ \mathrm{s} \approx 52\,\mathrm{m}$ long, which seems unreasonable.

A. Comment on ^{8}B specific configuration

$^{8}_{5}$B is a weakly bound nucleus on the proton-rich side of the drip line [26] with $Z = 5$ and $N = 3$ and the unpaired proton in the $1p_{3/2}$ orbit with only 137 keV g.s. (ground state) binding energy (compare roughly 4 MeV BE per nucleon we calculated with the von Weizsäcker formula as given in Krane pg. 68 [15]). The r.m.s. $1p_{3/2}$ radius has been estimated at 4.228 fm. Comparing with the SW (Saxon-Woods average nuclear potential) radius $R_0 = 2.4$ fm, the $1p_{3/2}$ odd proton, which will β^+ decay to a neutron to produce ${}^8_4$Be in about 770ms [27], is very weakly bound and is spatially extended (a *halo* more or less). One calculation [28] gives the proton separation energy for ${}^8_5$B as 0.74 MeV and that becomes negative -3.26 MeV if the calculation is repeated with $(Z, N) = (5, 2)$, i.e., if the proton-neutron ratio is increased by one a proton will "drip" off the nucleus immediately.

However, even the rare high energy 20.816 keV plasma proton would have a relatively large de Broglie wavelength of $\lambda = h/p \sim 200$ fm (see Section V below for a detailed calculation of the de Broglie wavelength). The target nucleus is $\lesssim 4$ fm. The incoming low energy proton would see a nucleus (and a nucleus with a strongly repulsive positive Coulomb field $\mathcal{O}(2\,\text{MeV})$ at that, as we discussed above) rather than individual nucleons, i.e., would not "see" the extended $1p_{3/2}$ proton.

The more fundamental question is, even with Coulomb barrier penetration could an incoming plasma proton actually "interrupt" a β^+ process?

V. NUCLEON SCALE CONSIDERATIONS

In β^+ decay (see Fig. 1), a virtual W^+ boson weak-force propagator connects an up to down quark transmutation to a e^+ and ν_e emission. The process is $p{:}udu \to n{:}ddu + e^+ + \nu_e$ where the W^+ is the propagator directed to the lepton emission vertex. Our notation $p{:}udu$ signifies *nucleon:quarks*, e.g., a proton and its constituent valence up, down and up quarks.[8]

If we simply apply the energy-time Uncertainty Principle, i.e.,

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

interpreted as, per *Concepts of Modern Physics; Arthur Beiser* [30], "an event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of the event does not exceed $\hbar/2\Delta E$", we may estimate the virtual W^+ lifetime in the β^+ decay process

[8] You could say that the W^+ couples to valence quarks at production, for our process $u \to d + e^+ + \nu_e$, and to $e^+\nu_e$ pairs at decay (β^+), $p \to n + e^+ + \nu_e$. [29].

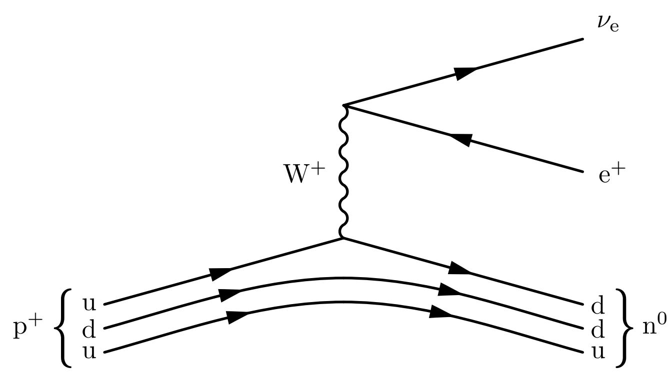

FIG. 1. β^+ decay, $p \to n + e^+ + \nu_e$, showing underlying proton to neutron valence quark change, $u \to d$, and virtual W^+ intermediate vector boson emission and decay to lepton pair e^+ and ν_e.

as

$$\tau_W \approx \frac{\hbar}{2M_W} = \frac{(6.582\ \times 10^{-16}\ \text{eV s})}{(2)(80.385\ \times 10^9\ \text{eV})} = 4.0941\ \times 10^{-27}\ \text{s}$$

where 80.385 GeV is the average on-shell W mass given in the PDG 2016 reference cited earlier from combined Tevatron and LEP experiments using the electron and muon decay modes. The W in these measurements was created in high energy c.m. (center of mass frame) electron positron collisions at LEP and Tevatron, i.e., it was not a virtual particle.

If the neutrino is emitted at the speed of light during that $\Delta t = \tau_W$, the packet length would be $\Delta t \cdot c = (4.0941\ \times 10^{-27}\ \text{s}) \cdot (2.9979\ \times 10^{10}\ \text{cm s}^{-1}) = 1.2274\ \times 10^{-16}$ cm.

We have from PDG 2016 an average width (on-shell) for the W^+ of $\Gamma_W = 2.085$ GeV (particle listings pg 615; average of Tevatron and LEP results). We can obtain a lifetime from that with the relation:

$$\tau = \frac{h}{\Gamma} = \frac{(4.1357 \times 10^{-24}\ \text{GeVs})}{(2.085\ \text{GeV})} = 1.9778 \times 10^{-24}\ \text{s}$$

Were that the Δt constraining the neutrino emission time the packet could be $5.9465\ 10^{-14}$ cm. We take note again that the W is not on-shell in a β^+ decay, but is rather a virtual boson.

However, we know that the wave packet describing a particle cannot be smaller than its De Broglie wavelength [15], which for a $p = 10$ MeV neutrino (e.g., a ^{8}B neutrino) would be

$$\lambda = c\frac{h}{p} = 2.9979\ 10^{10}\ \text{cm/s}\frac{4.1357 \times 10^{-15}\ \text{eV s}}{10\ \times 10^6\ \text{eV}} = 1.2398\ 10^{-11}\ \text{cm} \qquad \text{ultra-relativistic neutrino}$$

We can check for any significant increase in the virtual W^+ lifetime in nuclear β^+ decay resulting from the Lorentz factor for a typical nucleon velocity:

$$p_{nuc} \approx R^{-1} \quad \text{typical nucleon momentum inverse nuclear radius}$$
$$p_{nuc} \approx \hbar c\,(R)^{-1} = 197 \text{ MeV fm}/2.5 \text{ fm} = 78.93 \text{ MeV}$$
$$E_{nuc} = \sqrt{p_{nuc}^2 + m_{nuc}^2} = \sqrt{(78.93 \text{ MeV})^2 + (938 \text{ MeV})^2} = 941.59 \text{ MeV}$$
$$\gamma_{nuc} = E_{nuc}/m_{nuc} = 1.0035 \quad \text{Lorentz factor}$$

The increase in virtual lifetime attributable to time dilation in the frame of the decaying nucleon within the nuclear environment would be insignificant with $\gamma \sim 1$. The inverse nuclear radius approximation for typical nucleon momentum was given in [31].

The highest Fermi momentum level in the Fermi gas model is around $p_{Fermi} \sim 253$ MeV/c calculated for the ^{8}B nucleus. The $1p_{3/2}$ proton in this nucleus may well be at or above this higher momentum value. Proton knock-out experiments, in particular, exclusive scattering where the scattered $\mathcal{O}(5\,\text{GeV})$ accelerator beam electron and two ejected nucleons from a target nucleus are measured in the final state, $(e, e'pN)$ where N is a second proton or neutron emitted back-to-back with the first, have revealed that only about 80% of the contained nucleons scatter with a momentum conforming to single nucleons moving under the influence of an effective mean-field potential within the nucleus (i.e., the shell model or independent particle model).

Instead, $\sim$ 20% of the nucleons are short range correlated (SRC) pairs, predominantly neutron-proton systems (which have the same quantum numbers as the deuteron, $S = 1$ and $T = 0$) with very large relative momentum (up to a few times the Fermi momentum) but small center of mass momentum. You might say these pairs were "doing the neutron dance" as American pop group *The Pointer Sisters* sang, racing around and past each other, dangerously close as it were. See [17] for a well-written paper describing the experimental and theoretical details of a growing body of evidence that the internal quark structure of bound nucleons differs from that of free nucleons and that the SRC pair phenomenon is connected to, or responsible for, the EMC effect for valence quarks, that is, the reduction in the DIS (deep inelastic scattering) cross-section ratios for nuclei relative to deuterium.

This SRC phenomenon suggests to us that our ^{8}B $1p_{3/2}$ proton (or any of the solar β^+-decay protons in other nucleuss) is frequently SRC-paired with a neutron at very high momentum $p_{n,p} > p_{Fermi} > 253$ MeV/c and therefore even less likely (than when it is conforming to mean field approximations and more typical but nonetheless high momentum relative to plasma energies as we discussed above) to be affected by the low $\mathcal{O}$(eV) energies of plasma electrons or protons in the Sun, i.e., unlikely to be interrupted by plasma collisions while emitting a neutrino during the process of β^+ decay. The γ factor did not increase significantly for $p_{Fermi} \sim 253$ MeV/c vs the 78.93 MeV value used earlier, i.e., the increase in virtual lifetime attributable to time dilation in the frame of the decaying nucleon within the nuclear environment would still be insignificant with $\gamma \sim 1.0357$.

VI. TERRESTRIAL REACTOR EXPERIMENTS PROVIDE LOWER BOUND PACKET LENGTH

As Kayser pointed out in 2010 (cited earlier), the fact that the KamLAND experiment (c. 2002) observed oscillations at baselines of $\mathcal{O}$(100 km) implies that neutrino packets from β-decay of nuclear reactor fission fragments are $\sigma_x \geq 10^{-3}$ $\mathring{A}$, or $\geq 10^{-11}$ cm. We assume CP invariance (invariance of charge conjugation and parity combined), so our discussion of antineutrinos $\bar{\nu}_e$ is applicable to neutrinos ν_e, i.e., in general $P(\nu_\ell \to \nu_{\ell'}) = P(\bar{\nu}_\ell \to \bar{\nu}_{\ell'})$, $\ell, \ell' = e, \mu, \tau$.

You may use Eq. (1) and solve for σ_x given the energy and known baseline over which the neutrinos appeared coherent given that oscillations were observed. Per *Reactor-based neutrino oscillation experiments* by Bemporad *et al* (in *Reviews of Modern Physics, Volume 74, April 2002*) figure 2 (reactor flux and interaction spectrum), we will use antineutrino energy $E = 4$ MeV, i.e., around the peak energy of the reactor antineutrino spectrum. We will use Kayser's $\mathcal{O}$(100 km) for the average distance from the KamLAND detector to nearby nuclear reactors (some sources estimate $\langle L \rangle = 175$ km but we will use Kayser's value in order to replicate his σ_x estimate).

$$
\begin{aligned}
L_{coh} &= \frac{\sigma_x}{\Delta v} = \sigma_x \frac{2E^2}{\Delta m^2} \\
\sigma_x &= L_{coh} \Delta v \\
&= (100 \ \times 10^3 \mathrm{m}) \frac{\Delta m^2}{2E^2} \\
&= (10^5 \mathrm{m}) \frac{7.5 \ \times 10^{-5} \ \mathrm{eV}^2}{(4 \ \times 10^6 \mathrm{eV})^2} \\
\sigma_x(KamLAND) &= 2.3437 \ \times 10^{-3} \ \mathring{A} \\
&\geq 2.3437 \ \times 10^{-11} \ \mathrm{cm}
\end{aligned}
$$

A similar calculation based on an estimate of $L_{COH} \approx 10$ km for the Daya Bay reactor experiments at antineutrino energy 4 MeV gives us an estimate of the antineutrino packet length of $\geq 7.46 \times 10^{-11}$ cm [32]. Note that this calculation required that we use the Δm^2_{13} mass split of 2.445×10^{-3} eV^2 relevant in this short baseline reactor antineutrino context, rather than the $\Delta m^2_{12} = 7.5 \ \times 10^{-5}$ eV^2 solar split which is primarily the context of our discussions.

For the $p = 4$ MeV KamLAND reactor neutrino the de Broglie wavelength is $\lambda = h/p = 3.0996 \times 10^{-11}$ cm. These estimates based on terrestrial oscillation observations seem to agree on $\mathcal{O}\left(10^{-11} \text{ cm}\right)$ minimum packet length, supported by general de Broglie wavelength considerations. In light of the constraints on the lifetime of the virtual W boson above, i.e., the neutrino emission time interval should be not much greater than 10^{-27}-10^{-24} s, we suggest that the packet length of neutrinos emitted in β-decays are all $\mathcal{O}(10^{-11}$ cm).

VII. CODE FOR NUCLEAR RATE CALCULATIONS

The following is the `Python` computer code that implements the nuclear reaction rate integration equation Eq. (5) discussed in Section IV). Use assumes you have `SciPy`, `NumPy`, and `Python 3` and know how to make code adjustments if you have different versions:

```
# coding: utf-8
from scipy import constants
import math
```

```
import numpy as np

from scipy.integrate import simps

m_p_amu = constants.value('proton mass in u')
m_e_amu = constants.value('electron mass in u')
# Avogadro's number 6.022140857e+23
N_0 = constants.value('Avogadro constant')
c = constants.value('speed of light in vacuum') # m/s

# specify projectile and target charges
Z_1 = 1 # an incoming plasma proton
B8_Z = 5 # 8B nucleus has 5 protons
Z_2 = B8_Z # nucleus about to have a collision
# K, temperature at r=.0460, times 10^6 K (i.e., MK)
T_MK = 14.9420

# the reduced mass in atomic unit code
A_1 = m_p_amu  # A_1 incoming nucleon is a proton of mass 1.00727647.. u
# 8/5 nuclide mass per Krane Intro Nuclear Phys table of nuc props
B8_atomic_mass = 8.024606 # atomic mass units u, must subtract 5 electron mass
B8_electron_shell_mass = (B8_Z*m_e_amu)
B8_nuc_mass = B8_atomic_mass - B8_electron_shell_mass
# ignore positive correction for lost electron binding E, O(1e-6)
A_2 = B8_nuc_mass
mu_hat = (A_1 * A_2)/(A_1 + A_2)


# NACRE II equation 3 from
# http://arxiv.org/abs/1310.7099v1
# first take care of the constants out front of integral
```

```
# convert our operating temperature from million K to billion K as required
# mu_hat reduced mass u calculated earlier above; T_9 is global used below also
T_9 = T_MK / 1e3
in_front = 3.73e10 * mu_hat**(-1/2) * T_9**(-3/2)

# get Gamow peak; uses temperature in MK; Adelberger 1998 eq 2
# https://arxiv.org/abs/astro-ph/9805121
E0_keV = 1.2204*( Z_1**2 * Z_2**2 * mu_hat * T_MK**2  )**(1/3)

# construct array of integrand function values to call in the integration
# make array of dE steps, original limits around E0; the step is returned->dE
# we will use Adelberger 1998 pg 6 typical E solar fusion 5 kev - 30 keV
NACRE_low_lim_MeV = 0.005 # 5 keV in units of MeV
NACRE_high_lim_MeV = 0.030 # 30 keV in units of MeV
E_steps, dE = np.linspace(NACRE_low_lim_MeV, NACRE_high_lim_MeV, num=100, retstep=True)

# create simulated astrophysical factor S

# let us use simply the geometrical cross section, pi R^2
# Be7 to 8B astrophys reaction.pdf gives 2.84 fm for this weakly bound system
eff_radius = 2.84

S0_18_fudged  = math.pi * (eff_radius*1e-15)**2 # result in m^2

# now per Krane a measured cross section is multiplied by E of measurement
# to give S factor; we want our S factor to come to max at E = E0, so
# let us say the cross section was measured there
S0_18_fudged = S0_18_fudged * E0_keV # now have keV m^2

# we want the cross section sigma(E) with E in MeV and sigma in b
# so let us convert S0_17 to Mev b here
# times 1e28 converts m^2 to barn; divide by 1e3 if convert keV to Mev
```

```
S0 = (S0_18_fudged*1e28) / 1e3 # convert keV m^2 to MeV b

eta_frontend = 0.1575*Z_1*Z_2 # get constant part of eta calculation
def E_sigmaE(Energy):
    """calculate and return E * sigma(E) given E in MeV"""
    S_of_E = S0
    eta_of_E = eta_frontend * (mu_hat/Energy)**(1/2)
    return(S_of_E*math.exp(-2*math.pi*eta_of_E))

# returns E sigma(E) = S exp(-2pi eta) defined by Adelberger 1998 pg 6, eq 7



# will use T_9 global temperature in units 10^9 K
# this function populates the integrand array
def integrand_step(Energy):
    """accept E in MeV and calculate the integrand function value """
    return(math.exp(-11.605*Energy/T_9)*E_sigmaE(Energy))



# we provide f(E) as precalculated
# array integrand_fcn_values, E values in E_steps
iteration_limit = int(E_steps.shape[0])
integrand_fcn_values = np.zeros(iteration_limit)

for indx_1219 in range(0, iteration_limit):
    integrand_fcn_values[indx_1219] = integrand_step(E_steps[indx_1219])



integration_14GK = simps(  integrand_fcn_values, x=E_steps )

# and finish the calculation by applying the front-end to the integration
N_A_sigmav_8Bp_14MK = integration_14GK * in_front
```

```
# divide by Avogadro number to see sigma v (cross section * velocity)
sigmav_8Bp_14MK = N_A_sigmav_8Bp_14MK/N_0

# at solar r = 0.0460 number density of electrons ( same ~protons, neutral)
n_e_r0460 = 5.4670e25 # cm^-3
# don't know 7Be, so use 3He as proxy (same radius, from same SSM data)
n_3He_r0460 = 3.4914e20

# 16.7% of the 3He fuses with alpha's to produce 7Be
Be7_from_3He = n_3He_r0460 * 16.7 * 1e-2

# 0.12% of that 7Be captures a proton to from 8B (briefly)
B8_from_7Be = Be7_from_3He * 0.12 * 1e-2

collision_freq_8Bp = sigmav_8Bp_14MK * n_e_r0460 * B8_from_7Be
collision_interval_8Bp = 1 / collision_freq_8Bp


# parameters used, results
print("Temperature {0:.4f} GK".format(T_9))
print("reduced mass (mu hat): {0:.8f} u".format(mu_hat))
print("8B atomic mass (Krane): {0:.6f} u".format(B8_atomic_mass))
print("8B atom electron mass: {0:.6f} u".format( B8_electron_shell_mass ))
print("8B nuclear mass minus electron mass: {0:.6f} u".format(A_2))

print("electron mass: {0:.6f} u".format(m_e_amu))
print("proton mass: {0:.7f} u".format(m_p_amu))

print("simulated S factor for 8B p collisions: {0:.4f} keV b". format(S0*1e3))
```

```
print("NACRE II E0 (Gamow peak) = {0:.4f} keV".format(E0_keV))
print("calculated N_A <sigma v> 8B->p: {0:.4e} cm^3 mol^-1 s^-1".\
 format(N_A_sigmav_8Bp_14MK))
print("integrated over E range {0:.3f} to {1:.3f} keV".\
 format(NACRE_low_lim_MeV*1e3 ,NACRE_high_lim_MeV*1e3))
print("calculated sigma v: {0:.4e} cm^3 s^-1".format(sigmav_8Bp_14MK))

print("3He density: {0:.4e} cm^-3".format(n_3He_r0460))
print("7Be density: {0:.4e} cm^-3".format(Be7_from_3He))
print("8B resulting density: {0:.4e} cm^-3".format(B8_from_7Be))
print("nucleon projectile density: {0:.4e} cm^-3".format(n_e_r0460))
print("estimated collision frequency p on 8B: {0:.4e} s^-1".\
 format(collision_freq_8Bp))
print("estimated collision interval p on 8B {0:.4e} s".\
 format(collision_interval_8Bp))
# estimate neutrino packet length if constrained emission on that interval
neutrino_packet_8Bp = c*collision_interval_8Bp
print("corresponding neutrino packet len c*t then {0:.4f} m".\
 format(neutrino_packet_8Bp))


# accessory code: cross check integration reaction rate using
# equations 6 and 7 per Longland 2010 (T_MK*1e-3 is T_9)
# http://arxiv.org/abs/1004.4136v1
tau_Longland_0512 = 4.2487*(Z_1**2 * Z_2**2 * mu_hat * (T_MK*1e-3)**(-1))**(1/3)
# our simulated S0 is in MeV b as required by Longland equation
N_A_sigma_v_Longland = ( 4.339e8 / (Z_1*Z_2) ) * (1/mu_hat) * S0 *\
 math.exp(-tau_Longland_0512) * tau_Longland_0512**2
print("cross check above integrated result:")
print("tau: {0:.4f}".format(tau_Longland_0512))
print("NA sigma v: {0:.4e} cm^3 mol^-1 s^-1".format(N_A_sigma_v_Longland))
```

```
# accessory code:
# eta sommerfeld parameter calculator
print("reduced mass: {0:.4f} u".format(mu_hat))
print("target charge: {0:.1f}".format(Z_2))
E_eta = E0_keV/1e3 # they want MeV for eta_s formula
print("energy: {0:.4f} keV".format(E0_keV))
# eta_S for Sommerfeld parameter; equation from NACRE II paper
# http://arxiv.org/abs/1310.7099v1
eta_S = 0.1575*Z_1*Z_2*(mu_hat / E_eta)**(1/2)
print("eta at Gamow peak E0: {0:.4f}".format(eta_S) )
print("eta * 2 * pi = {0:.4f}".format(eta_S*2*math.pi))
# per Adelberger 1998 2pi eta should be gte 1 for WKB approx validity
# Gamow penetration factor then:
print("tunneling probability: {0:.4e}".format( math.exp(-eta_S*2*math.pi)  ))
```

VIII. READING AND WORKING WITH SSM DATA

The SSM (Standard Solar Model) data we have used in this paper were kindly provided in simple character tabular format by researcher Aldo Serenelli at SSM Data. The formal paper describing that model (and comparing to others) is found in [33] and an excellent description of solar models and the relevant concepts at [34]. The file header (the commented lines preceding the rows and columns of data) for the `GS98 comp neutrino and e profile.dat` [9] gives the format:

```
# Distribution of neutrino fluxes in the B16(GS98) standard solar model model.
#
# arXiv:1611.09867
#
```

[9] Do not be intimidated by the `dat` file extension, it is an ASCII text data file, albeit without MS Windows CR LF terminations, so on a Windows machine open (right click“open with”) in the Firefox browser to read it yourself rather than in Notepad.

```
# Columns in the Table below represent:
#
# 1) Radius of the zone in units of one solar radius
# 2) Temperature in units of 10^6 deg (K)
# 3) Logarithm (to the base 10) of the electron density in units of
#   cm^{-3}/N_A, where N_A is the Avogadro number
# 4) Mass of the zone with the given radius in units of one solar mass
# 5) Fraction of pp neutrinos produced in the zone
# 6) Fraction of pep neutrinos produced in the zone
# 7) Fraction of hep neutrinos produced in the zone
# 8) Fraction of beryllium 7 neutrinos produced in the zone
# 9) Fraction of boron 8 neutrinos produced in the zone
# 10) Fraction of nitrogen 13 neutrinos produced in the zone
# 11) Fraction of oxygen 15 neutrinos produced in the zone
# 12) Fraction of florine 17 neutrinos produced in the zone
#
# FORMAT: (f6.4,tr1,f6.3,tr1,f5.3,tr2,9(e14.8,tr2))
```

You should index columns in an array beginning at zero in most computer languages, so remember to decrement each column number above when looking for the desired column, e.g., the radius field will be in column zero in your array (more on loading to follow). The `FORMAT` line appears to be a `FORTRAN` file format specifier. For example, `tr1` is an instruction to tab one character to the right to the next position in the data record. We ignore this and simply rely on `NumPy` text loading function to recognize whitespace as data separators (delimiters) and LF as end of line characters. The following line of `Python` will load the SSM data file into a local `ndarray`. This assumes that you have loaded `NumPy` (as `np`) and are running `Python 3` (see VII for the import statements):

```
# read in the B16GS98 neutrino data set
B16GS98neutrinoData = np.loadtxt("GS98 comp neutrino and e profile.dat")
```

Following that load, you can use `NumPy` inscrutable slice syntax to get data entries of interest (in an interactive session for example), remembering to decrement column field identifiers from header:

```
# column 2 is the density; want column 2 of row 91
# make a scalar out of the single entry
GS98_e_dens_val  = B16GS98neutrinoData[91:92,2:3].flatten()[0]
# rem that [0] gets the single resulting value out
```

From the data header above you see that we just obtained the electron density at that data row 91. This is the row for data at solar radius $r_\odot = 0.0460$. If you have the data file open in Firefox browser (or are in a Linux environment) you can just read the file yourself and look for the radius of interest, the first column in each data row. The rows may wrap around in your display because of their length but the radius entries are recognizable as a series beginning at $r_\odot = 0.0005$, incrementing by 0.0005 each row, and ending at $r_\odot = 0.5000$. There is no need to give neutrino production data past that the midway point on the way out of the Sun as the temperature and density do not support the necessary nuclear reactions.

That electron density value, $N_e(normed) = 1.9580$ (`Python` variable `GS98_e_dens_val`), we just obtained is, as the header information told us, the base 10 log of the electron number density per cm^3 divided by Avogadro's number. To see the actual number density n_e then, $n_e = (10^{N_e(normed)})(N_0)$ where N_0 is Avogadro's number:

```
n_e_r0460 = 10**(GS98_e_dens_val)*N_0
print("Electron number density at r=0.0460 is {0:.4e} cm^-3".\
      format(n_e_r0460))
```

That should print, in an interactive session,

```
Electron number density at r=0.0460 is 5.4670e+25 cm^-3
```

You can load in columns of interest into a `NumPy` array and then work somewhat more sensibly with array indexing:

```
# get copy of B8 neutrino data col 8 as slice converted to array
B8_neut_data = np.asarray(  B16GS98neutrinoData[:,8:9] )
```

With the ^{8}B neutrino production fraction data in hand we could find the maximum emission point:

```
# get the index of the maximum entry
loc_B8_max = np.argmax(B8_neut_data)
print("row {0:.2f}".format(loc_B8_max))
```

In your `Python` interactive console (we are actually working in the somewhat more deluxe *Jupyter Notebook* environment where we can type notes and LaTeX mathematics and run code in the same browser page) you should see something like:

```
row 91.00
```

Be aware that the retrieved value for the ^{8}B flux fraction produced at that radial location, `B8_neut_data[91]=18.360`, is a normalized quantity $\Phi_j(r)$ derived from the actual flux as a function of the fractional radius (the normalized radius values we have been using, $r_\odot = 0.0005$ the core and the halfway point headed radially out of the Sun $r_\odot = 0.5000$):

$$\Phi_j(r) \equiv (1/F_j)\; df_j(r)/dr$$

where f_j (subscript j represents the particular solar neutrino species) is the flux as a function of radius in cm^{-2}s^{-1} and F_j is the total flux for this neutrino type (equation from [11]; total fluxes given in a separate data file at SSM Data or in [33] Table 6). So the absolute flux in a particular zone (the concentric spherical shells of thickness $r_2 - r_1$ where, as we said above, r_2 will always be $0.0005 + r_1$) should be $\Phi_{abs}(r) = F \int_{r-dr}^{r} \Phi_{norm}(r)\; dr$ (we dropped the subscript j for clarity; it is understood we are referring to one particular species). For example, if we integrate the entire column of normalized flux data for ^{8}B with the dr the radius fraction from the entire column of radius position data, we should obtain the published SSM total flux for this species, $\Phi(^8\mathrm{B}) = 5.46\, \times 10^6\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}$:

```
# get copy of 8B neutrino data col 8 as slice converted to array
B8col = 8
B8_neut_data = \
np.asarray(  B16GS98neutrinoData[:,int(B8col):int(B8col+1)] )

# get copy of neutrino data col 0 (radius) as slice converted to array
neut_dat_radius = np.asarray(  B16GS98neutrinoData[:,0:1] )

# want 1-d row array for the NumPy trapezoidal integrator fcn
# (simpler than axis specifications)
y_vals_to_int = np.reshape(B8_neut_data, B8_neut_data.shape[0])
dx_vals = np.reshape(neut_dat_radius, neut_dat_radius.shape[0])
```

```
# y is Phi_norm(r) and x is dr at each r
integrated_flux_fractions_B8 = np.trapz(  y_vals_to_int, x=dx_vals )

phi_B8_from_GS98df = phi_B8_SSM * integrated_flux_fractions_B8

print("flux fractions integrated: {0:.4f}".\
      format(integrated_flux_fractions_B8))
print("Abs flux total: {0:.4e}  cm^-2 s^-1".format(  phi_B8_from_GS98df  ))
```

The result printed on your console should be (note that all of the SSM neutrino species normalized flux series should integrate to 1):

```
flux fractions integrated: 0.9999
Abs flux total: 5.4596e+06  cm^-2 s^-1
```

The ^{8}B $\Phi_{norm}(r)$ conforms almost identically to a Maxwell distribution

$$f(x) = \sqrt{\frac{2}{\pi}} \frac{x^2 e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma^3}$$

where $\mu = -0.00022851$ and $\sigma = 0.03172368$ (we fit the SSM data using the `SciPy` non-linear least squares fitter `curve_fit` on the Maxwell function above with σ ("scale") and μ ("loc") as the parameters to fit and the flux data the dependent variable, the radius fractions the independent variable). If we wanted to know the absolute flux originating at our maximum emission zone at $r_\odot = 0.0460$ for this species we could integrate the Maxwell function fit at index 91 (using the `SciPy` `quad` integrator adapted from the Fortran libray `QUADPACK`):

```
# SSM 8B total flux prediction
phi_B8_SSM = 5.46e6 # cm^-2 s^-1

# use globals for loc and scale to avoid passng args to quad integrator
Mw_constant = np.sqrt(2/np.pi)
loc_Mw = -0.00022851 # popt[0] from the Maxwell fit, loc parameter
scale_Mw = 0.03172368 # popt[1] from the Maxwell fit, scale parameter
def f_Maxw( x ):
```

```
    return \
(Mw_constant*( x**2 *np.exp( -(x-loc_Mw)**2/(2*scale_Mw**2)))/scale_Mw**3)

# select radial slice location index of interest
index_08131052 = 91 # the data row number, 0 - 999
up_lim = dx_vals[index_08131052]
low_lim = dx_vals[index_08131052-1]
# do not attempt to integrate the core zero index, for obvious reasons

# quad integrates f from lower to upper limit of integration

integral_res_08131109, abserr = quad(f_Maxw, low_lim, up_lim)
# apply the absolute flux factor to integration result
abs_flux_08131110 = integral_res_08131109 * phi_B8_SSM
print("abs flux: {0:.4e} cm^-2 s^-1".format(abs_flux_08131110))
```

Your console result, the absolute ^{8}B neutrino flux originating at this zone, should be

```
abs flux: 4.9956e+04 cm^-2 s^-1
```

You could simply integrate the data at this zone directly (we used the `SciPy` Simpson's rule integrator for sample data in this case):

```
x_rad_upper = 91 # data row index, corresponds to a radial location

# Python range is to one less than upper index
dx_radius_simps = dx_vals[x_rad_upper-1:x_rad_upper+1]

integ_res_08121626 = \
simps(  y_vals_to_int[x_rad_upper-1:x_rad_upper+1], x=dx_radius_simps)

F_r_08121608 = (integ_res_08121626)*phi_B8_SSM
print("absolute flux at this slice: {0:.4e} cm^-2".format(F_r_08121608))
```

```
print(dx_radius_simps)
print( y_vals_to_int[x_rad_upper-1:x_rad_upper+1])
end{verbatim}

The console result should be
\begin{verbatim}
absolute flux at this slice: 4.9958e+04 cm^-2
[ 0.0455  0.046 ]
[ 18.2393535  18.3601275]
```

That is reasonably close agreement between the Maxwell fit integration vs the direct data integration. Use of the SSM neutrino flux data should be more clear at this point.

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[1] S. Nussinov, "Solar Neutrinos and Neutrino Mixing," Physics Letters **B63**, 201–203 (1976).

[2] Jörn Kersten and Alexei Yu. Smirnov, "Decoherence and oscillations of supernova neutrinos," [hep-ph] (2016), pp. 4,6, 1512.09068v2.

[3] A. A. Michelson, "On the Broadening of Spectral Lines," The Astrophysical Journal **II** (1895).

[4] George W. Collins, *Fundamentals of Stellar Astrophysics* (Online, 2003) pp. 350, 369, 371, author made 1989 published text available online.

[5] George B. Arfken and Hans J. Weber, *Mathematical Methods for Physicists, sixth edition* (Elsevier Academic Press, 2005) pp. 940–941.

[6] C. Patrignani *et al*, "Review of particle physics," Chin. Phys. C **40**, 100001 (2016), chapter 14, Neutrino mass, mixing and oscillations and other chapters for constants, methods, etc.

[7] R. Kippenhahn and A. Weigert, *Stellar Structure and Evolution* (Springer-Verlag, 1989) pp. 107–117,137–144.

[8] G. Kirchhoff and R. Bunsen, "Chemical Analysis by Spectral Observations," (1860) D. Brace translation of Pogg. Ann. Vol. 110. 1860 for collection Laws of Radiation and Absorption, 1901.

[9] Herman Haken and Hans Christoph Wolf, *The Physics of Atoms and Quanta* (Springer-Verlag, 2000) pp. 2–3, 114, 297, 301, 174, 294–295, translated by William D. Brewer.

[10] Roger D. Blandford and Kip S. Thorne, *APPLICATIONS OF CLASSICAL PHYSICS* (2013) authors frequently make various editions available online at Caltech (see link provided here,

but note this is not a permalink, so a search at this Caltech site may be required.

[11] Ilídio Lopes and Sylvaine Turck-Chièze, "SOLAR NEUTRINO PHYSICS OSCILLATIONS: SENSITIVITY TO THE ELECTRONIC DENSITY IN THE SUN'S CORE," [astro-ph.SR] (2013), See Fig. 1 and caption for SSM neutrino flux normalization., 1302.2791v1.

[12] S. Chandrasekhar, "Stochastic Problems in Physics and Astronomy," Reviews of Modern Physics **15**, 86–87 (1943), Appendix VII. Distribution of nearest neighbor in random distribution of particles.

[13] A. S. Richardson, "2019 NRL PLASMA FORMULARY," (2019).

[14] John Bahcall, "The ^{7}Be Solar Neutrino Line: A Reflection of the Central Temperature Distribution of the Sun," [astro-ph] (1994), astro-ph/9401024v1.

[15] Kenneth S. Krane, *Introductory Nuclear Physics* (John Wiley and Sons, 1988).

[16] Y. Xu *et al*, "NACRE II: an update of the NACRE compilation ...," (2013), update for charged-particle-induced thermonuclear reaction rates for nuclei $A < 16$, 1310.7099v1.

[17] Or Hen *et al*, "Nucleon-Nucleon Correlations, Short-lived Excitations, and the Quarks Within," [nucl-ex] (2017), 1611.09748v4.

[18] K Wiesemann, "A Short Introduction to Plasma Physics," (2004).

[19] Martin Asplund *et al*, "The chemical composition of the Sun," [astro-ph.SR] (2009), 0909.0948v1.

[20] "Atomic Spectra Database," National Institute of Standards and Technology site makes available, if the user is tenacious.

[21] W. C. Martin and W. L. Wiese, "Atomic Spectroscopy," Originally published as Chapt. 10 in Atomic, Molecular, and Optical Physics Handbook 1996, Drake.

[22] Eric G. Adelberger *et al*, "Solar fusion cross sections," Reviews of Modern Physics **70**, 1271 (1998), arXiv eprint link provided.

[23] H. A. Bethe , "Energy production in stars," (1967), Nobel Lecture by Hans Bethe December 11, 1967.

[24] S.B. Dubovichenko *et al*, "New results for the $p^7\mathrm{Be} \rightarrow^8 \mathrm{B}\gamma$ astrophysical s-factor ...," Elsevier Science B. V. (2019), arXiv eprint provided for doi.org/10.1016/j.nuclphysa.2018.11.033, 1811.03737.

[25] C. A. Bertulani and T. Kajino, "Frontiers in Nuclear Astrophysics," [nuc-ph] (2016), 1604.03197v2.

[26] K. Bennaceur *et al*, “Study of the ^{7}Be$(p, \gamma)^8$B and ^{7}Li$(n, \gamma)^8$Li capture reactions . . .,” [nucl-th] (1999), nucl-th/9901060v3.

[27] G. Audi *et al*, “NuBase 2012 evaluation of nuclear properties,” Chin. Phys. C **36** (2012).

[28] C. Samanta P. Roy Chowdhury, “Modified Bethe-Weizäcker mass formula with isotonic shift and new driplines,” [nucl-th] (2005), nucl-th/0405080v4.

[29] C. Rubbia, “Experimental observation of the intermediate vector bosons W^+, W^-, and Z^0 ,” (1985), Carlo Rubbia 1984 Nobel Prize lecture published also in Reviews of Modern Physics, 1985.

[30] Arthur Beiser, *Concepts of Modern Physics* (McGraw-Hill, 2003).

[31] K. Langanke and G. Martínez-Pinedo, “Nuclear weak-interaction processes in stars,” [nuc-th] (2002), 0203071v2.

[32] Kirk T. McDonald, “(Non)decoherence in the Daya Bay Antineutrino Experiment,” (2016).

[33] Nuria Vinyoles *et al*, “A NEW GENERATION OF STANDARD SOLAR MODELS,” [astro-ph.SR] (2016), dataset downloadable at www.ice.csic.es/personal/aldos, 1611.09867v4.

[34] Aldo Serenelli, “Alive and well: a short review about standard solar models,” [astro-ph.SR] (2016), 1601.07179v1.